International Grammar School

Year 12
Mathematics Extension 1
HSC Trial Examination
2011

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this question paper
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value
- Start each question in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Total Marks – 84
Attempt Questions 1 - 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Factorise completely $x^4 + 2x^3 - 2x^2 - 4x$ **2**

(b) Find the exact value of $\cos 15^\circ$. **2**

(c) The interval joining the points $A(9,12)$ and $B\ (1,b)$ is divided internally in the ratio 2:1 by the point $(a,0)$. Find *a* and *b*. **3**

(d) Solve $\dfrac{3}{x-2} \le 4$ **3**

(e) Find $\displaystyle\int \frac{dx}{\sqrt{36 - x^2}}$ **2**

Question 2 (12 marks) Use a separate writing booklet **Marks**

(a) If α , β and γ are the roots of $x^3 - 4x + 1 = 0$ find:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta\gamma$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **1**

(b) (i) Write $\sin x - \sqrt{3}\,\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise solve the equation $\sin x - \sqrt{3}\cos x = 1$ for $0 \le x \le 2\pi$. **2**

(c) $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line *k* is parallel to the tangent at *P* and passes through the focus *S* of the parabola.

(i) Show that the equation of the line *k* is $y = tx + a$ **2**

(ii) The line *k* intersects the *x*-axis at the point *Q*. Find the coordinates of the midpoint, *M*, of the interval *QS.* **2**

(iii) Find the equation of the locus of *M.* **1**

Question 3 (12 Marks) Use a separate writing booklet **Marks**

(a) Let $f(x) = e^{x+2}$

(i) Explain why $f(x)$ has an inverse function. **1**

(ii) Find the inverse function $f^{-1}(x)$ **1**

(iii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same number plane. Show all the important features. **2**

(b) (i) Show that $e^x - 2x^2 = 0$ has a root in the interval $2 < x < 3$. **2**

(ii) One approximate solution of the equation $e^x - 2x^2 = 0$ is $x = 2.5$. Use one application of Newton's method to find another approximation to this solution. Give your answer correct to three decimal places. **2**

(c) (i) Prove that $\dfrac{\sec^2 x}{\tan x} = \dfrac{\text{cosec}\, x}{\cos x}$ **2**

(ii) Use the substitution $u = \tan x$ to evaluate in exact form **2**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos ecx}{\cos x}.dx$$

Question 4 (12 Marks) Use a separate writing booklet **Marks**

(a) The polynomial $P(x) = x^3 + ax + b$ has $(x-5)$ as a factor and has a remainder of -60 when divided by $(x+5)$. **3**
Find the values of a and b.

(b) Use the method of mathematical induction to prove that $4^n + 8$ is divisible by 6 for $n \geq 1$. **3**

(c) (i) Sketch the graph of $y = 3\sin^{-1}\frac{x}{2}$, stating its domain and range. **3**

(ii) Show that the area of the region bounded by $y = 3\sin^{-1}\frac{x}{2}$, the x axis and the line $x = 1$ is given by $A = \frac{\pi}{2} + 3\sqrt{3} - 6\, units^2$. **3**

Question 5 (12 Marks) Use a separate writing booklet **Marks**

(a) Calculate the acute angle between the lines $x - 3y - 2 = 0$ and $x - 2y = 0$ to the nearest degree. **2**

(b) A particle is moving in a straight line such that its position at a time t is given by the equation

$$x = 2\cos\left(3t + \frac{\pi}{3}\right)$$

(i) Show that it is undergoing simple harmonic motion. **2**

(ii) State the period and amplitude of motion. **2**

(c) A bottle of water has a temperature of 20°C and is placed in a refrigerator whose temperature is 2°C. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water. This is expressed by the equation:

$$\frac{dT}{dt} = -k(T-2)$$

where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator.

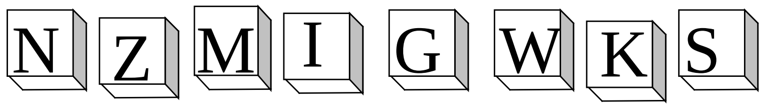
(i) Show that $T = 2 + Ae^{-kt}$ satisfies the equation. **1**

(ii) After 20 minutes in the refrigerator the temperature of the bottle of water is 10°C. What is the value of A and k in the above equation? **3**

(iii) How long will it take for the bottle of water to cool down to 5°C? Give your answer to the nearest minute. **2**

Question 6 (12 Marks) Use a separate writing booklet **Marks**

(a) In a word game, Carlo draws the 8 letter tiles shown below which he must arrange to make "words".

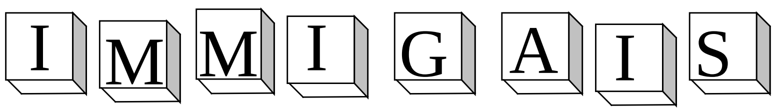


(i) In how many distinct ways can Carlo arrange the eight tiles in a line to make eight-letter "words"? **1**

(ii) Carlo randomly chooses four tiles from the eight. How many distinct ways can Carlo arrange the four tiles in a line to make four-letter "words"? **1**

(iii) Carlo randomly chooses four tiles from his 8 to swap with a second player. How many different groups of four tiles (in no particular order) can Carlo choose? **1**

(iv) After swapping with the second player Carlo has the eight tiles below. In how many distinct ways can Carlo arrange these eight tiles in a line to make eight-letter "words"? **1**



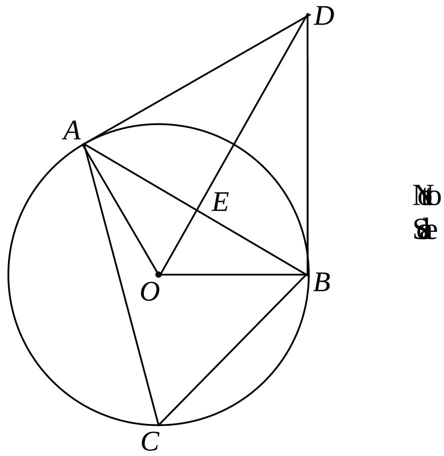
(v) In how many ways can the 8 tiles be arranged so that the "word" formed would start and end with M? **1**

(b) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} cos^2 2x \, dx$ as an exact value. **2**

Question 6 continues on the next page.

Question 6 continued. **Marks**

(c) The diagram shows points *A*, *B* and *C* on a circle centre *O*. Tangents are drawn from *A* and *B* which meet at *D*. *O* is joined to *D* and the interval *OD* intersects *AB* at *E*.



Copy or trace the diagram into your workbook.

(i) Prove that $\angle AOB = 2 \times \angle DAB$. **2**

(ii) Prove that *AOBD* is a cyclic quadrilateral. **1**

(iii) Prove that *E* is the midpoint of *AB.* **2**

Question 7 (12 Marks) Use a separate writing booklet **Marks**

(a) A ball is thrown from the origin *O* with a velocity *V* and angle of elevation of θ, where $\theta \neq \frac{\pi}{2}$. You may assume that

$$x = Vt\cos\theta \text{ and } y = -\frac{1}{2}gt^2 + Vt\sin\theta$$

where *x* and *y* are the horizontal and vertical displacements of the ball in metres from the *O* at time *t* seconds after being thrown.

(i) Let $h = \frac{V^2}{2g}$ and show the equation of flight of the ball is **3**

$$y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$$

(ii) The point of intersection when two balls are thrown with an angle of elevation of θ_1 and θ_2 is (a,b). **3**

Show that $a^2 < 4h(h-b)$.

(b) (i) Find all the solutions to the inequality $\frac{x}{1-x^2} \geq 0$ **3**

(ii) Show that $\tan x \sec x = \frac{\sin x}{1-\sin^2 x}$ **1**

(iii) Find all the solutions to $\tan x \sec x \geq 0$ when $0 \leq x \leq 2\pi$ **2**

END OF EXAMINATION

TABLE OF STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1} x^{n+1}, \ n \neq -1; x \neq 0, \text{if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x, \ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a} e^{ax}, \ a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \ a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \ a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \ a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \ a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \ \ x > 0$

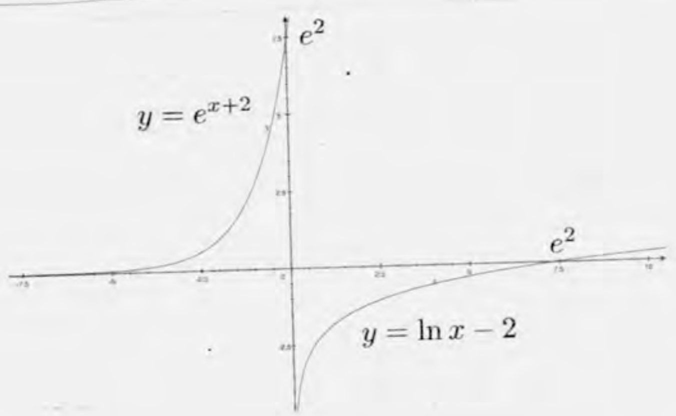
HSC Mathematics Extension 1 Yearly Examination

	Solution	Criteria
1(a)	$x^4+2x^3-2x^2-4x=x^3(x+2)-2x(x+2)$ $=(x^3-2x)(x+2)$ $=x(x^2-2)(x+2)$	2 Marks: Correct answer. 1 Mark: Finds one factor.
1(b)	$\cos(45-30)=\cos 45\cos 30+\sin 45\sin 30$ $=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$ $=\frac{1}{\sqrt{2}}(\frac{\sqrt{3}}{2}+\frac{1}{2})$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$	2 Marks: Correct answer. 1 Mark: Uses difference formula or states one correct exact ratio.
1(c)	$x=\frac{mx_2+nx_1}{m+n}$ $y=\frac{my_2+ny_1}{m+n}$ $a=\frac{2\times1+1\times9}{2+1}$ $0=\frac{2\times b+1\times12}{2+1}$ $=\frac{11}{3}$ $0=2b+12$ $b=-6$	3 Marks: Correct answer. 2 Marks: Correctly finds either *a* or *b*. 1 mark: Uses ratio division formula.
1(d)	$(x-2)^2\times\frac{3}{(x-2)}\le 4\times(x-2)^2$ $(x-2)3\le 4(x-2)^2 \quad x\ne 2$ $(x-2)(3-4x+8)\le 0$ $(x-2)(11-4x)\le 0$ $x<2$ and $x\ge\frac{11}{4}$	3 Marks: Correct answer. 2 Marks: Finds one correct solution. 1 Mark: Multiplies both sides of the inequality by $(x-2)^2$.
1(e)	$\int\frac{dx}{\sqrt{36-x^2}}=\sin^{-1}\frac{x}{6}+C$	2 Marks: Correct answer. 1 Mark: Recognises inverse sine function.

	QUESTION 2	
2(a) (i)	$x^3+0x^2-4x+1=0$ $\alpha+\beta+\chi=-\frac{b}{a}=-\frac{0}{1}=0$	1 Mark: Correct answer.
2(a) (ii)	$\alpha\beta\chi=-\frac{d}{a}=-\frac{1}{1}=-1$	1 Mark: Correct answer.
2(a) (iii)	$\alpha\beta+\alpha\chi+\beta\chi=\frac{c}{a}=\frac{-4}{1}=-4$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\chi}=\frac{\alpha\beta+\alpha\chi+\beta\chi}{\alpha\beta\chi}$ $=\frac{-4}{-1}$ $=4$	1 Mark: Correct answer.
b)(i)	If $\sin x - \sqrt{3}\cos x = R\sin(x-\alpha)$ then $R=\sqrt{1^2+(\sqrt{3})^2}$ and $\tan\alpha=\sqrt{3}$ $\therefore R = 2$ and $\alpha=\frac{\pi}{3}$ So $\sin x - \sqrt{3}\cos x = 2\sin\left(x-\frac{\pi}{3}\right)$	1 mark for R and 1 mark for α
b) (ii)	$\sin x - \sqrt{3}\cos x = 1$ $2\sin\left(x-\frac{\pi}{3}\right) = 1$ $\sin\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$ $x-\frac{\pi}{3}=\frac{\pi}{6},\frac{5\pi}{6}$ $x=\frac{\pi}{2},\frac{7\pi}{6}$	1 for writing in form $\sin\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$ 1 for solving for x

2(c) (i)	To find the gradient of the tangent $y = \frac{1}{4a}x^2$ $\frac{dy}{dx} = \frac{1}{2a}x$ At $P(2at, at^2)$ $\frac{dy}{dx} = \frac{1}{2a} \times 2at = t$ Line k has a gradient of t and passes through $S(0, a)$ $y - y_1 = m(x - x_1)$ $y - a = t(x - 0)$ $y = tx + a$	2 Marks: Correct answer. 1 Mark: Finds or states the gradient of the tangent at *P*.
2(c) (ii)	To find the coordinates of Q Substitute $y = 0$ into $y = tx + a$ then $x = -\frac{a}{t}$ or $Q(-\frac{a}{t}, 0)$ To find the coordinates of M $x = \frac{x_1 + x_2}{2} = \frac{-\frac{a}{t} + 0}{2} = -\frac{a}{2t}$ $y = \frac{y_1 + y_2}{2} = \frac{0 + a}{2} = \frac{a}{2}$ $M(-\frac{a}{2t}, \frac{a}{2})$	2 Marks: Correct answer. 1 Mark: Finds the coordinates of *Q*.
2(c) (iii)	To find the equation of the locus eliminate t. However y is independent of t. $y = \frac{a}{2}$	1 Mark: Correct answer.

	QUESTION 3	
3(a)(i)	$f'(x) = e^{x+2} > 0$ for all x, ie f(x) is monotonic increasing for all x and hence an inverse function exists Or for every value of y there is only one value of x, ie there is a 1-1 correspondance between y and x	1 mark for adequate explanation. Note that 'passes the horizontal line test" is not adequate without further explanation of what this means.
3(a) (ii)	$f(x) = e^{x+2}$ or $y = e^{x+2}$ Inverse function is $x = e^{y+2}$ $\log_e x = y + 2$ $y = \log_e x - 2$ $f^{-1}(x) = \log_e x - 2$	1 Mark: Correct answer.

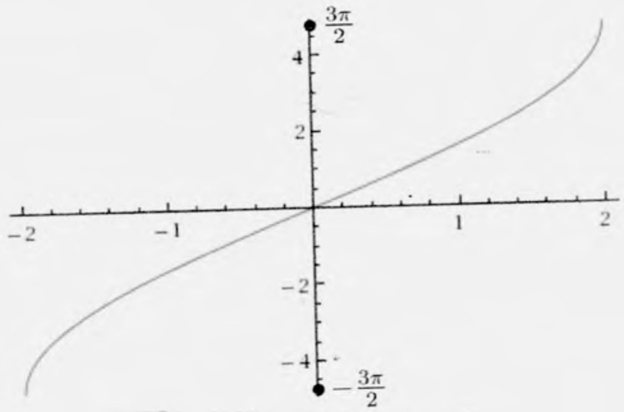
3(a) (ii)		2 Marks: Correct answer. 1 Mark: Draws one of the graphs correctly.
3(b)(i)	$f(x) = e^x - 2x^2$ $f(2) = e^2 - 2(2)^2 = -0.6109... < 0$ $f(3) = e^3 - 2(3)^2 = 2.0855... > 0$ Since sign change from x=2 to x=3 and f(x) is continuous in this interval (since x>0) then a root exists 2<x<3.	2	1 mark for showing change of sign 1 mark for statement of test. Must include continuity and why continuous.

3(b)(ii)	Root of $y = e^x - 2x^2$ using $x = 2.5$ $f(x) = e^x - 2x^2$ $f'(x) = e^x - 4x$ $f(2.5) = -0.318$ $f'(2.5) = 2.182$ $x_1 = 2.5 - \frac{f(2.5)}{f'(2.5)}$ $= 2.5 - \frac{-0.318}{2.182}$ $= 2.5 - (-0.145)$ $= 2.645$	2	1 for values of $f(2.5)$ and $f'(2.5)$ 1 use of N M to obtain answer.

3(c) (i)	$\text{LHS} = \frac{\sec^2 x}{\tan x}$ $= \frac{1}{\cos^2 x} \div \tan x$ $= \frac{1}{\cos^2 x} \times \frac{\cos x}{\sin x}$ $= \frac{1}{\cos x} \times \text{cosec } x$ $= \frac{\text{cosec } x}{\cos x}$ $= \text{RHS}$	2 Marks: Correct answer. 1 Mark: Makes significant progress towards the solution.
3(c) (ii)	$u = \tan x$ $u = \tan\frac{\pi}{3} = \sqrt{3}$ $u = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $du = \sec^2 x dx$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos ecx}{\cos x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$ $= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{u} du$ $= \left[\log_e u\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$ $= \log_e \sqrt{3} - \log_e \frac{1}{\sqrt{3}}$ $= \log_e 3$	2 Marks: Correct answer. 1 Mark: Recognises the use of part (i) or makes progress in the substitution.

QUESTION 4			
4(a)	$p(x) = x^3 + ax + b$ has $(x - 5)$ as one of its factors $\therefore \quad p(5) = 0$ $125 + 5a + b = 0$ Remainder = -60 when divided by $(x + 5)$. $\therefore \quad p(-5) = -60$ $-125 - 5a + b = -60$ Solve simultaneously to find *a* and *b*. $5a + b + 125 = 0 \quad ①$ $-5a + b - 65 = 0 \quad ②$ $2b + 60 = 0 \quad ① + ②$ $2b = -60$ $b = -30$ $5a - 30 + 125 = 0$ $5a = -95$ $a = -19$	3	1 mark for use of factor theorem 1 mark for use of remainder theorem 1 mark for solving simultaneously Part marks as appropriate if other approaches taken

b)	Step 1 show true of n = 1 $4^1 + 8 = 12$ 12 is divisible by 6 $\therefore$ true for $n = 1$ Step 2 Assume true for $n = k$ Ie $4^k + 8 = 6p$ (p is a positive integer) Step 3: hence prove true for $n = k + 1$ $4^{k+1} = 6q$ (q is a positive integer) $LHS = 4^{k+1} + 8$ $= 4(4^k) + 8$ $= 4(4^k + 8 - 8) + 8$ $= 4(4^k + 8) - 32 + 8$ $= 4(6p) - 24$ $= 24p - 24$ $= 6(4p - 4)$ $= 6q$ (q is positive integer > 1 since $p > 1$) $= RHS$ Conclusion Using the principle of induction Since true for $n = 1$, and since if true for $n = k$ is also true for $n = k + 1$, by induction is true for all $n \geq 1$.	4	1 mark for step 1 2 marks for step 3 Or 1 mark for significant progress or a simple error in step 3

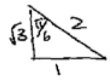
(c)(i)	$y = 3\sin^{-1}\dfrac{x}{2}$ Domain: $-2 \le x \le 2$ Range: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$  	1 mark each domain and range and correct graph

c)(ii)

Area under $y = 3\sin^{-1}\frac{x}{2}$ from $x=0$ to $x=1$.

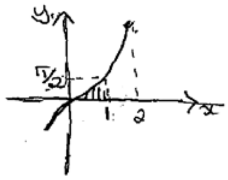
$x=0 \quad y=0$

$x=1 \quad y = 3\sin^{-1}\left(\frac{1}{2}\right)$

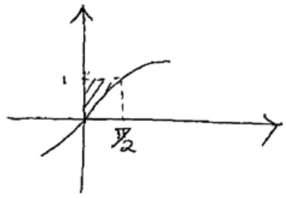


$= 3.\frac{\pi}{6}$

$= \frac{\pi}{2}.$



Inverse of $y = 3\sin^{-1}\frac{x}{2}$ is $y = 2\sin\frac{x}{3}$.



Area needed is the same as rectangle $1 \times \frac{\pi}{2} - \int_0^{\pi/2} 2\sin\frac{x}{3}.dx$.

$\therefore A = \frac{\pi}{2} - 2\int_0^{\pi/2} \sin\frac{x}{3}\,.dx$

$= \frac{\pi}{2} + 6\left[\cos\frac{x}{3}\right]_0^{\pi/2}$

$= \frac{\pi}{2} + 6\left[\cos\frac{\pi}{6} - \cos 0\right]$

$= \frac{\pi}{2} + 6.\frac{\sqrt{3}}{2} - 6$

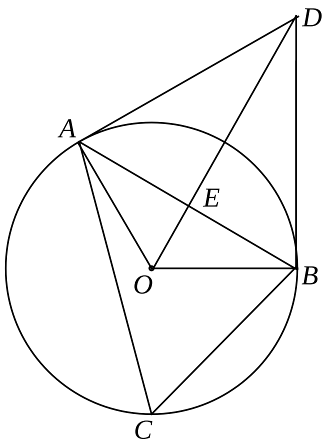
$= \frac{\pi}{2} + 3\sqrt{3} - 6.$

2 marks or 1 mark for evidence of working towards correct solution

1 mark.

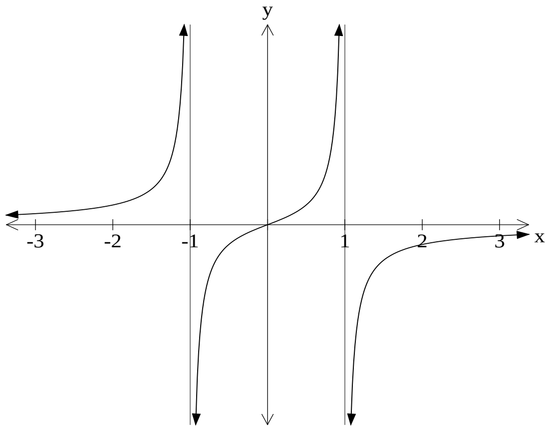
	QUESTION 5		
5(a)	$x - 3y - 2 = 0$ $x - 2y = 0$ $3y = x - 2$ $2y = x$ $y = \frac{x}{3} - \frac{2}{3}$ $y = \frac{x}{2}$ Gradient $= \frac{1}{3}$ Gradient $= \frac{1}{2}$. $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \cdot \frac{1}{3}} \right\rvert$ $= \left\lvert \frac{\frac{1}{6}}{\frac{7}{6}} \right\rvert$ $= \frac{1}{7}$ $\theta = \tan^{-1}\left(\frac{1}{7}\right)$ $= 8^\circ$ (nearest degree)	2	1 for gradients of the two lines 1 for evaluating the angle
b) i)	$x = 2\cos\left(3t + \frac{\pi}{3}\right)$ $\dot{x} = -2\sin\left(3t + \frac{\pi}{3}\right) .3$ $\dot{x} = -6\sin\left(3t + \frac{\pi}{3}\right)$ $\ddot{x} = -6\cos\left(3t + \frac{\pi}{3}\right) . 3$ $= -18\cos\left(3t + \frac{\pi}{3}\right)$ $\ddot{x} = -9\left(2\cos\left(3t + \frac{\pi}{3}\right)\right)$ $\ddot{x} = -3^2 x$ which is of the form $\ddot{x} = -n^2 x$ $\therefore$ particle is in SHM	2	1 mark for correct differentiations 1 mark for writing in the correct form and stating a conclusion
ii)	Period $= \frac{2\pi}{n}$ $= \frac{2\pi}{3}$ Amplitude =2 units	1 1	

5(c) (i)	$T = 2 + Ae^{-kt}$ or $Ae^{-kt} = T - 2$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T-2)$	1 Mark: Correct answer.
5(b) (ii)	Initially $t = 0$ and $T = 20$ $T = 2 + Ae^{-kt}$ $20 = 2 + Ae^{-k\times 0}$ $A = 18$ Also $t = 20$ and $T = 10$ $T = 2 + 18e^{-kt}$ $10 = 2 + 18e^{-k\times 20}$ $e^{-k\times 20} = \frac{8}{18}$ $-20k = \log_e \frac{4}{9}$ $k = -\frac{1}{20}\log_e \frac{4}{9}$ $= \frac{1}{20}\log_e \frac{9}{4}$ $= 0.04054651081...$	3 Marks: Correct answer. 2 Marks: Finds the value of *A* and an expression for *k*. 1 Mark: Finds the value of *A*.
5(b) (iii)	We need to find t when $T = 5$ $T = 2 + 18e^{-kt}$ $5 = 2 + 18e^{-kt}$ $e^{-kt} = \frac{3}{18}$ $-kt = \log_e \frac{1}{6}$ $t = \frac{1}{k}\log_e 6$ $= 20\frac{\log_e 6}{\log_e \frac{9}{4}}$ $= 44.19022583$ It will take about 44 minutes for the bottle to cool to 5ºC?	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

	QUESTION 6		
6 a) (i)	Ways of arranging 8 different tiles = 8! = 40 320	1	
(ii)	Ways of arranging 4 chosen from 8 = $^8\mathbf{P}_4$ = 1680	1	
(iii)	Ways of choosing 4 from 8 = $^8\mathbf{C}_4$ = 70	1	
(iv)	Ways of arranging 8 different tiles with 2 M's and 3 I's = $\frac{8!}{3! \times 2!} = 3\ 360$	1	
(v)	With M's placed at the ends, leaves 6 to arrange, with 3 I's. Ways of arranging 6 different tiles with 3 I's $= \frac{6!}{3!} = 120$	1	
6(b)	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 2x\, dx = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 1 + \cos 4x\, dx$ $= \frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= \left(\frac{\pi}{4} + \frac{1}{8}\sin 2\pi\right) - \left(\frac{\pi}{6} + \frac{1}{8}\sin\frac{4\pi}{3}\right)$ $= \frac{\pi}{4} + \frac{1}{8}\times 0 - \frac{\pi}{6} - \frac{1}{8}\times\frac{\sqrt{3}}{2}$ $= \frac{\pi}{12} - \frac{\sqrt{3}}{16}$ (= 0.154)	2	1 mark for obtaining integral 1 mark for correct substitution
c) (i)	 Aim prove $\angle AOB = 2 \times \angle DAB$ $\angle DAB = \angle ACB$ ($\angle$bet tangent & chord $=\angle$ in alt segment) $\angle AOB = 2 \times \angle ACB$ ($\angle$ at centre is twice $\angle$ at circ on same ar $\therefore \angle AOB = 2 \times \angle DAB$ (since $\angle DAB = \angle ACB$)	2	2 marks for complete proof. 1 mark if some relevant facts are stated or proof is incomplete

(ii)	$\angle DAO = \angle DBO = 90^\circ$(tangent perpendicular to rad $\therefore \angle DAO + \angle DBO = 180^\circ$ (sum of two right angles) $\therefore$ opposite angles of *AOBD* are supplementary $\therefore$ *AOBD* is a cyclic quadrilateral.	1	1 mark as long as statement that opposite angles are supplement ary, and why.
(iii)	Aim: Prove that *E* is the midpoint of *AB*. *AO* = *BO* (equal radii) *AD* = *BD* (tangents from an external point are equal in length.) $\therefore$ *AOBD* is a kite $\therefore$ *OD* bisects AB (symmetry of a kite) $\therefore$ E is midpoint of AB.	2	Can also be done by isosceles triangles 2 marks for complete proof. 1 mark if some relevant facts are stated or proof is incomplete

	QUESTION 7	
7(a) (i)	$x = Vt\cos\theta \quad (1)$ $y = -\frac{1}{2}gt^2 + Vt\sin\theta \quad (2)$ From eqn (1) $t = \frac{x}{V\cos\theta}$ sub into eqn (2) $y = -\frac{1}{2}g(\frac{x}{V\cos\theta})^2 + V(\frac{x}{V\cos\theta})\sin\theta$ $= -\frac{gx^2}{2V^2\cos^2\theta} + \frac{\sin\theta x}{\cos\theta}$ $= -\frac{gx^2\sec^2\theta}{2V^2} + \tan\theta x$ $= -\frac{2gx^2\sec^2\theta}{4V^2} + \tan\theta x$ $= -\frac{x^2\sec^2\theta}{4h} + \tan\theta x \quad$ Using $h = \frac{V^2}{2g}$ $= x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Makes *t* the subject of eqn (1) or equivalent progress.
7(a) (ii)	Now (a,b) satisfies the equation $y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta)$ $b = a\tan\theta - \frac{1}{4h}a^2(1+\tan^2\theta)$ $4hb = 4ha\tan\theta - a^2(1+\tan^2\theta)$ $(1+\tan^2\theta)a^2 - 4ha\tan\theta + 4hb = 0$ $a^2\tan^2\theta - 4ha\tan\theta + 4hb + a^2 = 0$ Quadratic equation has 2 solutions if the discriminate is greater than zero. $b^2 - 4ac > 0$ $(-4ha)^2 - 4a^2(4hb + a^2) > 0$ $16h^2a^2 - 16a^2hb - 4a^4 > 0$ $4a^2(4h^2 - 4hb - a^2) > 0$ $4h^2 - 4hb - a^2 > 0$ $a^2 < 4h^2 - 4hb$ $a^2 < 4h(h-b)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Substitutes (a,b) into equation of flight and simplifies.

7(b) (i)	$(1-x^2)^2 \times \frac{x}{1-x^2} \geq 0 \times (1-x^2)^2 \qquad x \neq \pm 1$ $(1-x^2)x \geq 0$ $(1-x)(1+x)x \geq 0$  From the graph $0 \leq x < 1$ and $x < -1$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds part of the solution or demonstrates some understanding.

7(c) (ii)	$\text{LHS} = \tan x \sec x$ $= \dfrac{\sin x}{\cos x} \times \dfrac{1}{\cos x}$ $= \dfrac{\sin x}{\cos^2 x}$ $= \dfrac{\sin x}{1 - \sin^2 x}$ $= \text{RHS}$	1 Mark: Correct answer.
7(c) (iii)	Using parts (i) and (ii) $0 \le \sin x < 1$ and $\sin x < -1$ $0 \le x < \dfrac{\pi}{2}$ No solution Solution also in the second quadrant $0 \le x < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < x \le \pi$	2 Marks: Correct answer. 1 mark: Makes some progress using parts (i) and (ii).